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MATHEMATICAL MODELING FOR CALCULATING THE FOURIER SERIES

Keywords: Mathematical modeling, Fourier series, periodic functions, variable, analysis, number series, practical applications, expression in feeling, mathematical theory, functional analysis.

Annotation: This article explores the calculation of the Fourier series using mathematical modeling methods. The Fourier series plays a crucial role in expressing periodic or non-periodic functions as a series. The paper introduces the fundamental concepts of the Fourier series, its mathematical theory, and its applications in real-world scenarios. As a result, we gain deeper insights into the properties of Fourier series and their practical applications.

Fourier series for odd and even functions. Let a function with a period T = 2π be given, that is, f(x + 2π) = f(x). The Fourier series and coefficients of the given function will be as follows:

$$f(x)=\frac{a_0}{2}+\underset{n=1}{}(a_n\cos nx+b_n\sin nx)$$

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cdot\cos nxdx$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cdot\sin nxdx$$

Below, we derive formulas for calculating the Fourier series coefficients of even and odd functions.

If the function f(x) is integrable on [–a; a], then

$$\int_{-a}^{a}f(x)dx=\int_{-a}^{0}f(x)dx+\int_{0}^{a}f(x)dx$$

By substituting x with -x in the second integral, we can apply it to (5):

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$$\int_{-a}^{0} f(x)dx = \int_{a}^{0} f(-x)d(-x) = -\int_{a}^{0} f(-x)dx = \int_{0}^{a} f(-x)dx$$

$$\int_{-a}^{a} f(x)dx = \int_{0}^{a} [f(x) + f(-x)]d(x),$$

If the function f(x) is odd, $f(-x) = -f(x)$

$$\int_{-a}^{a} f(x)dx = \int_{0}^{a} [f(x) - f(x)]d(x) = 0,$$

If the function f(x) is even, that is, $f(-x) = f(x)$

$$\int_{-a}^{a} f(x)dx = \int_{0}^{a} [f(x) + f(-x)]d(x) = 2 \cdot \int_{0}^{a} f(x)d(x)$$

The product of two even functions or two odd functions is an even function, while the product of an even and an odd function is an odd function. Considering (7), we calculate the Fourier series coefficients for even and odd functions:

1) Let the function f(x) be an even function with a period T = 2π that satisfies Dirichlet conditions on [-π, π].

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)dx = a_0 = \frac{2}{\pi} \int_{0}^{\pi} f(x)dx$$

$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \cos kxdx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cdot \cos kxdx$$

$$b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \sin kxdx = 0$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_k \cdot \cos kx$$

For the even function, the Fourier series consists solely of cosines, $b_k = 0$.

Let the function f(x) be an odd function with a period T = 2π that satisfies Dirichlet conditions on [-π, π].

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$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = 0$$

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot\cos kxdx = 0$$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot\sin kxdx = \frac{2}{\pi}\int_{0}^{\pi} f(x)\cdot\sin kxdx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} b_k\cdot\sin kx$$

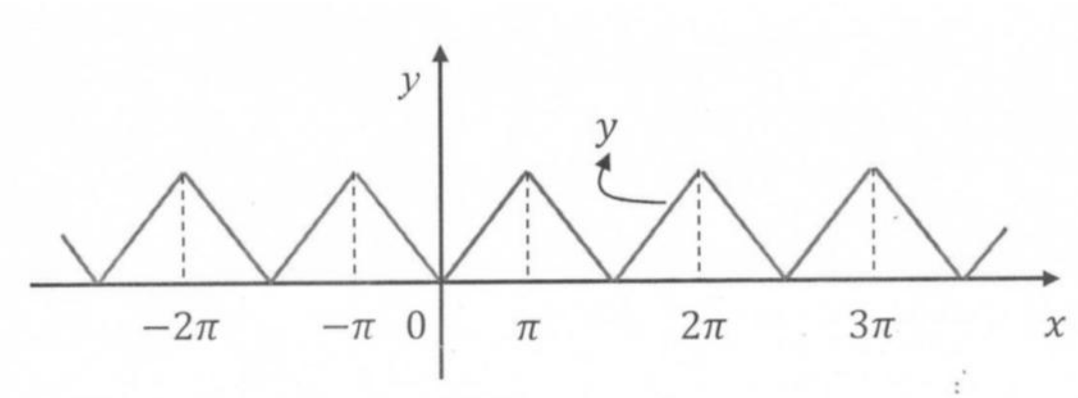
The Fourier series for the odd function consists solely of sines, $a_o = 0$, $a_k = 0$.

Example:Series of a function with a period T = 2π

$$f(x) = \begin{cases} -x, & agar \quad x\in(-\pi,0) \\ x, & agar \quad x\in[0,\pi) \end{cases}$$

Expand the Fourier.

Solution: The even function satisfies Dirichlet conditions on the interval (-π, π) (see Figure 1)



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$$a_0 = \frac{2}{\pi}\int_{0}^{\pi} f(x)dx = \frac{2}{\pi}\cdot\frac{x^2}{2}\Bigg|_0^{\pi} = \pi,$$

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$$a_k = \frac{2}{\pi}\int_0^{\pi} f(x)\cdot\cos kx dx = \frac{2}{\pi}\left(\frac{x\cdot\sin kx}{k}+\frac{\cos kx}{k^2}\right)\Bigg|_0^{\pi} =$$

$$\frac{2}{\pi}\cdot\frac{1}{k^2}\cdot(\cos k\pi-\cos 0) = \frac{2}{\pi}\cdot\frac{1}{k^2}\cdot\left[(-1)^k-1\right] =$$

$$=\begin{cases} -\dfrac{4}{k^2\cdot\pi}, & k \ \ toq \ \ bo'lsa, \\ 0 \quad , & k \ \ juft \ \ bo'lsa. \end{cases}$$

$$f(x) = \frac{\pi}{2}-\frac{4}{\pi}\cdot\sum_{k=1}^{\infty}\frac{\cos(2k-1)x}{(2k-1)^2} = \frac{\pi}{2}-\frac{4}{\pi}\left(\cos x+\frac{\cos 3x}{3^2}+\frac{\cos 5x}{5^2}+\ldots\right)$$

$$x=0, \qquad 0 = \frac{\pi}{2}-\frac{4}{\pi} \quad 1+\frac{1}{3^2}+\frac{1}{5^2}+\cdots+\frac{1}{(2n-1)^2}+\cdots$$

$$1+\frac{1}{3^2}+\frac{1}{5^2}+\cdots+\frac{1}{(2n-1)^2}+\cdots = \frac{\pi^2}{8}$$

$$S = 1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2}+\cdots = ?$$

$$S = \left(1+\frac{1}{3^2}+\frac{1}{5^2}+\cdots+\frac{1}{(2n-1)^2}+\cdots\right)+\left(\frac{1}{2^2}+\frac{1}{4^2}+\cdots+\frac{1}{(2n)^2}+\cdots\right)$$

$$S = \frac{\pi^2}{8}+\frac{1}{2^2}\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2}+\cdots\right)$$

$$S = \frac{\pi^2}{8}+\frac{1}{4}S$$

$$\frac{3\cdot S}{4} = \frac{\pi^2}{8} \qquad S = \frac{\pi^2}{6}.$$

Calculating the Fourier series using mathematical modeling plays an important role in studying the periodic properties of functions and expressing them. The Fourier series is widely used in industry and technology for obtaining complex waves or signals. With this method, functions can be computed, and their analytical properties revealed, applying mathematical theories. According to the results, mathematical modeling methods ensure great efficiency not only theoretically but also in practice. In the future, a deeper study of the Fourier series and exploration of its new applications are required. By creating mathematical models, the understanding of problems and processes in solving them can be simplified. Additionally, the development of these methods opens new opportunities for specialists and scientists.

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