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EVALUATIAN OF MEASUREMENT UNCERTAINTY USING THE MONTE CARLO SIMULATION MODELING METHOD

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Annotation: This article examines the application of the Monte Carlo simulation modeling method for the evaluation of measurement uncertainty. The Monte Carlo simulation approach is presented as an effective method that enables the assessment of uncertainty in measurement results using random variables. The article identifies the primary sources of uncertainty in measurements and describes the stages of the modeling process. Compared to traditional methods, Monte Carlo simulation modeling allows for high-accuracy results while maintaining sensitivity to the complexity of the measurement model and the distribution forms of uncertainties. The findings confirm that this method is an effective tool for improving measurement quality and achieving reliable uncertainty evaluation.

Keywords: Measurement uncertainty, Monte Carlo simulation modeling, statistical modeling, measurement model, random variables, sources of uncertainty, result reliability.

Introduction

The accuracy and reliability of measurement results hold critical importance in many areas of modern science and industry. Every measurement process inherently involves a certain degree of uncertainty. Identifying and evaluating this uncertainty is essential for improving the quality of the measurement system and ensuring the reliability of decision-making.

So, what is measurement uncertainty, and how does it affect the resulting value? Measurement uncertainty is a parameter associated with the result of a measurement that characterizes the dispersion of values that could reasonably be attributed to the measurand. This implies that measurement uncertainty is used to express the degree of variability (or dispersion) in measurement results.

In 1993, with the aim of establishing a unified approach to uncertainty evaluation, eight international organizations and the national metrology institutes of 32 countries jointly published the Guide to the Expression of Uncertainty in Measurement (GUM). Alongside the adoption of GUM, empirical approaches to uncertainty evaluation were also in use. Between 1993 and 1994, the ISO 5725 series of standards (Parts 1 through 6) were successively published, further contributing to methodological development in this area. Traditional methods of uncertainty evaluation, particularly those based on the GUM methodology, are widely used. However, in complex and nonlinear measurement models, the GUM-based approach may not always provide sufficient accuracy. Therefore, alternative methods, including Monte Carlo Simulation Modeling (MCM), are gaining prominence.

Monte Carlo simulation modeling relies on statistical analysis and enables the modeling of the distribution of uncertainty sources using random variables, allowing for the estimation of resultant

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measurement uncertainty. This method makes it possible to obtain highly accurate results regardless of the complexity of the measurement model or the level of variability involved. A more detailed look at the Monte Carlo method reveals that it originated in the mid-20th century during World War II and was initially developed to solve problems in nuclear physics. The method was pioneered by Polish-American mathematician **Stanisław Ulam**, renowned physicist and mathematician **John von Neumann**, and physicist-mathematician **Nicholas Metropolis**, all of whom contributed significantly to its development. In the 1940s, during the development of the atomic bomb at the **Los Alamos National Laboratory** in the United States, scientists faced the need to perform computations based on random events. While recovering from illness, Stanisław Ulam conceived the idea of probabilistic modeling while thinking about a card game (specifically, solitaire).
Later, the method was named "Monte Carlo" — a reference to the city of Monte Carlo in Monaco, famed for its casinos and games of chance. Initially, the method was used to simulate the behavior of nuclear particles and to evaluate complex integrals. With the advent of computers, the method experienced rapid development, as Monte Carlo simulations typically require thousands of repeated calculations.

The Monte Carlo simulation modeling method is typically applied in the study of processes characterized by numerous sources of uncertainty and inherent randomness. In particular, it is used in the following areas:

- **Uncertainty evaluation in measurements**: The 2008 GUM Supplement 1 to the Guide to the Expression of Uncertainty in Measurement (GUM) specifically recommends the use of the Monte Carlo method for calculating measurement uncertainty. This method is especially useful when the measurement model cannot be solved analytically or is too complex for traditional methods.
- **Finance and economics**: Monte Carlo simulations are employed to assess risks, forecast asset values, and model complex investment scenarios.
- **Physics and engineering**: The method is used to analyze complex systems such as radiation transport, heat transfer, and structural integrity.
- **Chemistry and biology**: It is applied to simulate molecular motion, reaction rates, and uncertainties in microbiological processes.
- **Computer graphics**: Monte Carlo methods are used to approximate realistic light behavior, particularly in techniques such as ray tracing to enhance photorealism.

The primary reason for using this method is that, in highly complex systems where it is difficult to obtain an exact mathematical solution, it becomes necessary to conduct numerous random experiments and statistically analyze the outcomes. In this article, we will provide information on how the Monte Carlo method can be used to assess measurement uncertainty.

When is Monte Carlo simulation applied for uncertainty evaluation?

- **When uncertainty sources are complex and statistically random in nature:**
  If the calibration process involves several random factors affecting the measurements (e.g., temperature, pressure, human factors, internal noise of the instrument), then the probability distribution of each factor is determined. Monte Carlo simulation is then used to model the overall uncertainty resulting from these factors.

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- **When it is difficult or impossible to compute overall uncertainty using analytical formulas:**
  If the measurement model is very complex and the conventional uncertainty propagation formulas (as described in the Guide to the Expression of Uncertainty in Measurement – GUM) are impractical to apply, the Monte Carlo simulation can replicate the model thousands of times on a computer, generating a distribution of uncertainty.
- **When the uncertainty in the measurement model follows a non-normal distribution:**
  If the measurement exhibits uneven (asymmetric) or non-Gaussian distributions, Monte Carlo simulation takes such conditions into account and provides more accurate results.
- **When interrelated (correlated) factors are present:**
  The Monte Carlo method also allows for the consideration of correlated sources of uncertainty, which is important for real-world models.

To explain the Monte Carlo simulation method in simple terms, it involves solving complex problems by conducting many experiments using random numbers. For example, when it is difficult to determine an exact measurement result, we perform hundreds or thousands of "estimated" measurements. Each iteration yields slightly different results (since in practice, no two measurements are exactly the same). By analyzing the large number of results, we gain a clearer understanding of the overall trend (average value). In essence, Monte Carlo simulation does not rely on a single "attempt," but instead aggregates many trials and averages the results to approach the true value.

Let us consider a simple example: Suppose we throw a ball into the air and want to know how high it goes. However, each throw may vary slightly due to factors such as wind, the strength of the throw, and minor differences in the ball's weight. So, we throw the ball 100 times and record the height of each throw. Then, we calculate the average height and conclude: "on average, the ball reaches this height." This is Monte Carlo simulation. Typically, the ball reaches a height of 10 meters. Due to wind and variations in the force applied by the hand, the height may vary by ±1 meter each time. That is, each throw results in an approximate height between 9 m and 11 m.

Now, we obtain a random result.

Throw Number)	Height (meters)
1	9.5
2	10.3
3	9.8
4	10.1
5	9.7
6	10.2
7	10.0
8	9.6
9	10.4
10	9.9

Now we calculate the average height (arithmetic mean):

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1. First, we find the sum: 9.5+10.3+9.8+10.1+9.7+10.2+10.0+9.6+10.4+9.9=99.5

2. Then, we divide by 10: $\overline{h} = \frac{99,5}{10}$ = 9.95 meter After throwing the stone 10 times, the average height was found to be 9.95 meters.

- We determine the standard deviation for the measurement uncertainty.
- Then, we calculate the standard uncertainty of the mean value.

$$U((\overline{h}))=\frac{S}{\sqrt{n}}$$

Here, n = 10 (the number of stone throws).

We square the difference between each height and the average height:
We calculate:

Throw	Height (h)	$(\mathbf{h}-\mathbf{9.95})^2$
1	9.5	$(9.5-9.95)^2$= 0.2025
2	10.3	$(10.3-9.95)^2$ = 0.1225
3	9.8	$(9.8-9.95)^2$ = 0.0225
4	10.1	$(10.1-9.95)^2$ = 0.0225
5	9.7	$(9.7-9.95)^2$ = 0.0625
6	10.2	$(10.2-9.95)^2$ = 0.0625
7	10.0	$(10.0-9.95)^2$ = 0.0025
8	9.6	$(9.6-9.95)^2$ = 0.1225
9	10.4	$(10.4-9.95)^2$ = 0.2025
10	9.9	$(9.9-9.95)^2$ = 0.0025

Now, we sum all the values:

Sum=0.2025+0.1225+0.0225+0.0225+0.0625+0.0625+0.0025+0.1225+0.2025+0.0025=0.8245

3. We calculate the variance: The formula for variance is:

$$S^2=\frac{(\mathrm{h}-\overline{h})^2}{n-1}=\frac{0.8245}{10-1}\approx \mathbf{0.0916}$$

4. We calculate the standard deviation:

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$S=\sqrt{0.0916} \approx$ **0.3027 metr**

5. The uncertainty of the mean value:

Results:

- Average height: 9.95 meters
- Standard deviation: 0.30 meters
- Monte Carlo uncertainty: 0.096 meters

Final result: (9.95 ± 0.096) meters

This example illustrates the measurement uncertainty in determining the flight height of a simple ball. In the Monte Carlo method, this process is considered significantly more complex, with the number of measurements typically ranging from 1,000 to 10,000, calculated through artificial simulation to achieve high accuracy. Programming techniques greatly facilitate the implementation of this process. Among the most popular programming languages used in the Monte Carlo method is Python. Why Python specifically?

1. Python is easy and fast to code.

- Monte Carlo involves generating and analyzing random data multiple times.
- Python's syntax is very simple, fast, and readable, making it ideal for computationally intensive methods like Monte Carlo simulations.

2. It has powerful libraries.

- numpy: generates large quantities of random numbers and performs fast mathematical operations.
- scipy: used for statistical and scientific calculations.
- matplotlib, seaborn: produce visually appealing graphical representations of results.
- pandas: facilitates convenient data manipulation.
  Monte Carlo simulations can be executed with just a few lines of code using Python libraries.

3. Computation speed can be accelerated.

- Python utilizes optimized internal libraries written in C.
- When working with numpy arrays, Python performs calculations very quickly.
- Even for large-scale simulations (e.g., 1,000,000 iterations), Python maintains efficient performance.

4. Widely adopted in the scientific community.

- Python is one of the most commonly used programming languages for Monte Carlo simulations in both academic and industrial projects.
- Consequently, numerous ready-made methods, examples, and libraries are available.

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In conclusion, the evaluation of measurement uncertainty often involves complex and stochastic conditions that require extensive measurement processes. Such processes may necessitate thousands or even tens of thousands of experimental measurements, each inherently subject to associated errors. These errors can significantly undermine the reliability of measurement results. The Monte Carlo simulation modeling method provides substantial assistance in addressing this problem.

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