

APPLICATIONS OF ANTISYMMETRIC POLYNOMIALS TO ELEMENTARY MATHEMATICAL PROBLEMS

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Annotation: this article explores a relatively under-researched yet important topic in the theory of polynomials within the course of Algebra and Number Theory - antisymmetric polynomials. It begins by presenting the concepts of symmetric and antisymmetric polynomials, their main properties, theorems, and lemmas. The article also discusses some applications of antisymmetric polynomials in elementary mathematics, particularly their role in factoring polynomials, with illustrative examples.

Keywords: antisymmetric polynomials, symmetric polynomials, theory of polynomials, algebra, number theory, polynomial factorization, elementary mathematics, theorem, lemma, properties of polynomials, antisymmetry.

Annotatsiya: mazkur maqolada algebra va sonlar nazariyasi kursining ko'phadlar nazariyasi bo'limiga oid bo'lgan kam o'rganilgan, ammo muhim mavzu - antisimmetrik ko'phadlar haqida so'z yuritiladi. Dastlab simmetrik va antisimmetrik ko'phadlar tushunchalari, ularning asosiy xossalari, teorema va lemmalari bayon qilinadi. Shuningdek, antisimmetrik ko'phadlarning elementar matematikadagi tatbiqlari, jumladan, ko'phadlarni ko'paytuvchilarga ajratishdagi o'rni misollar orqali ochib beriladi.

Kalit so'zlar: antisimmetrik ko'phadlar, simmetrik ko'phadlar, ko'phadlar nazariyasi, algebra, sonlar nazariyasi, ko'paytuvchilarga ajratish, elementar matematika, teorema, lemma, polinomlar xossalari, antisimetriya.

Аннотация: в данной статье рассматривается сравнительно малоизученная, но важная тема в разделе теории многочленов курса алгебры и теории чисел - антисимметричные многочлены. Сначала приводятся понятия симметричных и антисимметричных многочленов, их основные свойства, теоремы и леммы. Также рассматриваются некоторые приложения антисимметричных многочленов в элементарной математике, в частности, их роль в разложении многочленов на множители, с примерами.

Ключевые слова: антисимметричные многочлены, симметричные многочлены, теория многочленов, алгебра, теория чисел, разложение многочленов, элементарная математика, теорема, лемма, свойства многочленов, антисимметрия.

In the section on the theory of polynomials within the Algebra and Number Theory course, students are thoroughly introduced to symmetric polynomials - that is, polynomials that remain unchanged when any two of their variables are interchanged. The fundamental theorem on symmetric polynomials and its many applications are studied.

In this article, however, we will focus on a relatively less-studied class of polynomials - antisymmetric polynomials - which are very closely related to symmetric polynomials. We will introduce antisymmetric polynomials and consider some of their applications in elementary mathematics.

Antisymmetric polynomials are those that change their sign when any two of their variables are swapped.

Let us first consider antisymmetric polynomials in two variables. Examples of such polynomials include $x-yx-yx-y$, $x^3-y^3x^3-y^3x^3-y^3$, and $x^4y-xy^4x^4y-xy^4x^4y-xy^4$. For instance, if we swap the variables x and y in the polynomial $x^3-y^3x^3-y^3x^3-y^3$, we get $y^3-x^3y^3-x^3y^3-x^3$, which is equal to $-(x^3-y^3)-(x^3-y^3)-(x^3-y^3)$. Therefore, $x^3-y^3x^3-y^3x^3-y^3$ is an antisymmetric polynomial. In the same way, it can be shown that $x-yx-yx-y$ and $x^4y-xy^4x^4y-xy^4x^4y-xy^4$ are also antisymmetric polynomials.

As an example of an antisymmetric polynomial in three variables, consider the polynomial $(x-y)(x-z)(y-z)(x-y)(x-z)(y-z)(x-y)(x-z)(y-z)$. If we swap the variables x and y , we get $(y-x)(y-z)(x-z)=- (x-y)(x-z)(y-z)(y-x)(y-z)(x-z)=- (x-y)(x-z)(y-z)(y-x)(y-z)(x-z)=- (x-y)(x-z)(y-z)$. Likewise, if other pairs of variables are interchanged, the sign of the polynomial changes.

Here we present an important property of antisymmetric polynomials: the square of an antisymmetric polynomial is a symmetric polynomial.

In fact, when we interchange any two variables, an antisymmetric polynomial changes its sign. However, the square of the polynomial does not change its sign. Therefore, when we swap any two variables in the square of an antisymmetric polynomial, the polynomial remains unchanged-that is, the square of an antisymmetric polynomial is a symmetric polynomial.

Not only the square of an antisymmetric polynomial, but also the product of any two antisymmetric polynomials is a symmetric polynomial. This is because, when we swap any two variables, both of the polynomials change sign, so the overall sign of their product does not change.

On the other hand, if we multiply an antisymmetric polynomial by a symmetric polynomial, the result is an antisymmetric polynomial. This happens because, when we swap any two variables, the antisymmetric polynomial changes sign while the symmetric one does not, so the product changes sign overall.

Let us now clarify how any antisymmetric polynomial is constructed. The above statement points to a general method for constructing antisymmetric polynomials: we multiply an antisymmetric polynomial by any symmetric polynomial, and the product will be an antisymmetric polynomial.

A natural question arises: can we find a single antisymmetric polynomial such that, when multiplied by all symmetric polynomials, it generates all antisymmetric polynomials for a given number of variables?

Let's start with polynomials in two variables. In this case, the antisymmetric polynomial we seek is $x-yx-yx-y$. In other words, the following theorem holds.

Let us now present the main theorem and lemma about antisymmetric polynomials.

Theorem. Any antisymmetric polynomial $f(x,y)f(x,y)f(x,y)$ in two variables can be expressed as

$f(x,y)=(x-y)g(x,y), f(x,y)=(x-y)g(x,y), f(x,y)=(x-y)g(x,y)$, where $g(x,y)g(x,y)g(x,y)$ is a symmetric polynomial in x and y .

Lemma. If $f(x,y)f(x,y)f(x,y)$ is an antisymmetric polynomial, then $f(x,x)=0. f(x,x)=0. f(x,x)=0.$

In other words, when x and y coincide, an antisymmetric polynomial becomes zero.

Antisymmetric polynomials have applications in many areas of mathematics. Examples include polynomial factorization, calculation of the discriminant and checking the roots of equations, proving inequalities and identities, and rationalizing denominators.

Let us now consider some applications in elementary mathematics, in particular, applications to polynomial factorization. The fundamental theorem on antisymmetric polynomials makes it significantly easier to solve certain problems in elementary algebra. Since any antisymmetric polynomial in three variables is divisible by the polynomial $(x-y)(x-z)(y-z)$, it is always possible to factor an antisymmetric polynomial in this way. Here, the remaining factor is a symmetric polynomial. In turn, this symmetric polynomial can sometimes also be factored. It should be noted that, in order to find the ratio of an antisymmetric polynomial to this cubic polynomial, it is not practical to perform a direct polynomial division. Instead, using specific numerical values for the variables is a more convenient method. In particular, if the antisymmetric polynomial has degree three, the ratio will be a polynomial of degree zero—that is, a constant. The relation will be an affinity, that is, it will hold for arbitrary values of x, y, z . Therefore, to determine the number of x, y, z , it is sufficient to assign some (mutually distinct) numerical values to x, y, z in the last equation, from which the number of x, y, z is determined.

If $P(x, y, z)$ is a fourth-degree antisymmetric homogeneous form, then $P(x, y, z) / ((x-y)(x-z)(y-z))$ will be a first-degree homogeneous symmetric form, that is, $(ax+by+cz)$, and here it is sufficient to assign some numerical values to x, y, z in order to determine the unknown coefficients a, b, c .

Similarly, if $P(x, y, z)$ is a fifth-degree antisymmetric homogeneous form, then the ratio will be a second-degree homogeneous symmetric form, that is, $(ax^2+by^2+cz^2+2dxy+2exz+2fyz)$, where a, b, c, d, e, f are unknown coefficients. To find the two unknowns d, e, f , we need to assign numerical values to x, y, z twice.

If $P(x, y, z)$ is a sixth-degree antisymmetric homogeneous form, then $P(x, y, z) / ((x-y)(x-z)(y-z))$ will be a third-degree homogeneous symmetric form, that is, $(ax^3+by^3+cz^3+3dxy^2+3exz^2+3fyx^2+3fyz^2+3dxyz)$, where a, b, c, d, e, f are unknown coefficients. To find the three unknowns d, e, f , we need to assign numerical values to x, y, z three times.

References:

1. Ummatova, Mahbuba Axmedovna, and Abdurahim Tursunboyevich Mamatqulov. "AL-XORAZMIY ASARLARINING AMALIY AHAMIYATI HAQIDA." *INTERNATIONAL CONFERENCE DEDICATED TO THE ROLE AND IMPORTANCE OF INNOVATIVE EDUCATION IN THE 21ST CENTURY*. Vol. 1. No. 10. 2022.
2. Ummatova, Mahbuba Axmedovna, and Olimaxon Oxunjonovna Rahmonova. "ELEMENTAR MATEMATIKADA ANTISIMMETRIK KO'PHADLAR." *INTERNATIONAL CONFERENCE DEDICATED TO THE ROLE AND IMPORTANCE OF INNOVATIVE EDUCATION IN THE 21ST CENTURY*. Vol. 1. No. 10. 2022.
3. Ахмедовна, Умматова Махбуба. "РОЛЬ ЗАДАЧ ТЕОРИИ ЧИСЕЛ ПРИ ПОВЫШЕНИИ МАТЕМАТИЧЕСКИХ ЗНАНИЙ УЧАЩИХСЯ." *Ученый XXI века* 1-2 (2017).
4. Akhmedovna, Ummatova Makhbuba. "From the methodology of using nonstandard tests in the design of practical training on number theory." *Genius Repository* 27 (2023): 17-22.
5. Sulaymonov M.M., and Ummatova M.A.. "KASR TARTIBLI DIFFERENSIAL OPERATOR ISHTIROK ETGAN INTEGRO-DIFFERENSIAL TENGLAMALAR UCHUN INTEGRAL SHARTLI MASALALAR" *Экономика и социум*, no. 6-2 (109), 2023, pp. 526-530.
6. UMMATOVA, MAHBUBA. "BO'LINISH BELGILARI, SONLARNING UMUMIY BO'LUVCHISI VA KARRALI MAVZUSI BO'YICHA AMALIY MASHG'ULOTNI LOYIHALASHDA NOSTANDART TESTLARDAN FOYDALANISH METODIKASIDAN." *Scienceweb academic papers collection* (2023).

7. Nosirovich, Nosirov Sobirzhon, and Ummatova Makhbuba Ahmedovna. "AUTOMORPHISM OF NUMERICAL SYSTEMS." Open Access Repository .12 (2022): 197-201.
8. Ummatova, M. A. "DIDACTICAL AND PRACTICAL FUNCTIONS OF MATH CLASS." Galaxy International Interdisciplinary Research Journal 10.12 (2022): 259-262.
9. Умматова, М., Г. Ахмедова, and О. Махмудова. "Практическая направленность в обучении математике." Теория и практика современных гуманитарных и естественных наук. 2014.
10. Ahmedovna, Ummatova M., and Esonov M. Mukimjonovich. "Methodology of Performing Practical Independent Work." *JournalNX*, vol. 8, no. 12, 13 Dec. 2022, pp. 171-176, doi:10.17605/OSF.IO/YP2CD.
11. Axmedovna, Ummatova Mahbuba, and Ilhomjonova Shahnozaxon Ilhomjonovna. "TALIMDA BIOLOGIYA VA MATEMATIKA FANLARINING OZARO ALOQASI HAQIDA." BARQARORLIK VA YETAKCHI TADQIQOTLAR ONLAYN ILMIY JURNALI 2 (2022): 816-817.
12. Akhmedovna, Ummatova Makhbuba. "Teaching of Numerical Systems and Their Extensions." Genius Repository 27 (2023): 23-28.
13. Madraximov, Shuxratjon, and Maxfuza Madraximova. "Umumiy o 'rta ta'lim maktablarida fanlararo aloqadorlikning ahamiyati." MAKTABGACHA VA MAKTAB TA'LIMI JURNALI 3.5 (2025).
14. Madraximova, Maxfuza, and Shuxratjon Madraximov. "Umumiy o 'rta ta'lim maktablarida adabiyot fanini o 'qitishda zamonaviy texnologiyalar integratsiyasi." MAKTABGACHA VA MAKTAB TA'LIMI JURNALI 3.5 (2025).
15. Shuhratjon, Madraximov, and Madraximova Mahfuza. "SUN'IY INTELLEKT TIZIMLAR HAQIDA." INTERDISCIPLINE INNOVATION AND SCIENTIFIC RESEARCH CONFERENCE. Vol. 2. No. 20. 2024.
16. Shukurovich, Madrahimov Shuhratjon, and Madrahimova Mahfuza Ahmedovna. "Measures For Monitoring And Evaluation Of Power Activity In Higher Education." *JournalNX*: 423-426.
17. Shuhratjon, Madraximov, and Madraximova Mahfuza. "TEXNOLOGIK TA'LIMDA FANLARARO ALOQADORLIK." INTERDISCIPLINE INNOVATION AND SCIENTIFIC RESEARCH CONFERENCE. Vol. 2. No. 20. 2024.
18. Shukurovich, Madraximov Shuxratjon, and Madraximova Maxfuza Axmedovna. "INFORMATIKA TA'LIMIDA SAMR TEXNOLOGIYASIDAN FOYDALANISH." ZAMONAVIY TA'LIMDA RAQAMLI TEXNOLOGIYALARDAN FOYDALANISH ISTIQBOLLARI 1.1 (2024): 151-153.
19. Sattorova, D. Yu. "The use of Modern Educational Technologies in Teaching Physics." AMERICAN JOURNAL OF SOCIAL AND HUMANITARIAN RESEARCH. ISSN 26909626.
20. Dilshoda, Sattorova. "Dictated Games in Primary Education as an Important Factor in Guiding Students to Creative Thinking." *JournalNX*, vol. 7, no. 03, 2021, pp. 163-166.
21. Sattorova, D. "USING CROSSWORD PUZZLES IN PHYSICS LESSONS." ASIA PACIFIC JOURNAL OF MARKETING & MANAGEMENT REVIEW ISSN: 2319-2836 Impact Factor: 8.071 11.12 (2022): 32-34.
22. Sattorova, D. "IMPORTANCE OF MODERN EDUCATIONAL TECHNOLOGIES IN TEACHING PHYSICS IN PART OF "ELECTRICITY AND MAGNETISM". Science and innovation 2.B10 (2023): 214-218.
23. Sattorova, D., and Sh Jo'Martova. "Using Modern Educational Methods, Determining Students' Mastery Level." *JournalNX*, vol. 8, no. 12, 24 Dec. 2022, pp. 509-511, doi:10.17605/OSF.IO/M948B.
24. Shukurovich, Madraximov Shuxratjon, and Madraximova Maxfuza Axmedovna. "INFORMATIKA FANIDAN DARSLARNI KUZATISH VA TAHLIL QILISH METODIKASI." ZAMONAVIY

- TA'LIMDA RAQAMLI TEXNOLOGIYALARDAN FOYDALANISH ISTIQBOLLARI 1.1 (2024): 188-190.
25. Ahmedovna, Ummatova M., et al. "Logic as A Science on the Forms and Laws of Right Thinking." *JournalNX*, vol. 9, no. 5, 12 May. 2023, pp. 40-42, doi:10.17605/OSF.IO/4RXUB.
 26. Axmedovna, Ummatova Mahbuba. "USING ANTI-SYMMETRICAL PONYMS IN SOLVING ELEMENTARY MATHEMATICS PROBLEMS." *Galaxy International Interdisciplinary Research Journal* 11 (2023): 507-509.
 27. Axmedovna, Ummatova Maxbubaxon, and Savriddinova Madinaxon. "MATEMATIKANI O'QITISHNI TAKOMILLASHTIRISHDA MANTIQUIY TESTLARDAN FOYDALANISH." *INTERDISCIPLINE INNOVATION AND SCIENTIFIC RESEARCH CONFERENCE*. Vol. 2. No. 15. 2023.
 28. Maxbubaxon, Ummatova, Muydinjanov Davlatjon, and Nematov Bexruzbek. "AFFIN KOORDINATALAR SISTEMASIDA NUQTANI KOORDINATA O 'QLARIGA NISBATAN SIMMETRIK BO 'LGAN NUQTANING KOORDINATALARINI TOPISH USULLARI." *INNOVATIVE DEVELOPMENTS AND RESEARCH IN EDUCATION 2* (2023): 329-333.
 29. Ummatova, Mahbuba, and Temurbek Yoqubov. "TALABALAR MUSTAQIL ISHLARI VA UNI BAJARISHNI O 'RGATISH METODIKASIDAN." *Talqin va tadqiqotlar* 1 (2023).
 30. UMMATOVA, MAXBUBA. "MATEMATIKANI O'QITISHDA ZAMONAVIY AXBOROT TEXNOLOGIYALARIDAN FOYDALANISH." *Scienceweb academic papers collection* (2023).