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MATHEMATICAL MODEL AND DYNAMIC ANALYSIS OF THE PMSM

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Annatation. In this paper, a mathematical model of the Permanent Magnet Synchronous Motor (PMSM) drive is developed and analyzed.
The proposed model is designed to accurately describe the dynamic behavior of the PMSM, evaluate power losses, and facilitate control system modeling.
Based on the dynamic equations of the PMSM in the synchronous reference frame (dq0), the state-space equations are derived, where iron losses—particularly eddy current (Foucault) losses—are modeled by means of a resistance R_c connected in parallel with the magnetizing branch.
In addition, analytical expressions for copper (Cu) losses and iron losses under steady-state conditions are presented. The developed model can be effectively used for analyzing the efficiency of PMSM drives, performing numerical simulations, and optimizing control systems.

Keywords: Permanent Magnet Synchronous Motor (PMSM), dynamic model, dq0 transformation, synchronous reference frame, electromagnetic torque, iron losses, eddy current, power efficiency, mathematical model.

Mathematical Model of PMSM

To perform a theoretical analysis of the Permanent Magnet Synchronous Motor (PMSM) drive, it is necessary to develop a mathematical model that can accurately describe the machine's behavior. From a structural point of view, the PMSM is very similar to the conventional synchronous motor, and therefore its dynamic model is based on the classical synchronous machine equations.

When the machine is described in the abc phase reference frame, the obtained equations are nonlinear and time-dependent. However, when expressed in the dq rotating reference frame, the equations become much simpler, which increases computational speed and facilitates control system modeling. In the steady-state operation, the variables remain constant over time, further simplifying the analysis.

The dq model associated with the rotor is the most widely used and reliable model in the scientific literature. It neglects magnetic saturation, eddy current, and hysteresis losses and assumes that the back electromotive force (EMF) is sinusoidal. These simplifications allow predicting the PMSM's behavior with sufficient accuracy.

If the motor is designed for line-start operation, the rotor is equipped with a cage winding, similar to that of induction motors, and the model is accordingly modified. Furthermore, models that include iron losses—particularly eddy current losses—have also been developed, providing more accurate evaluations of the energy efficiency.

More complex models are developed using the Finite Element Method (FEM). This approach enables high-precision modeling of the PMSM's electromagnetic, thermal, and magnetic flux distribution processes. However, due to their complexity and high computational requirements, the simulation time of such models is typically long.

Dynamic Model. To describe and analyze the Permanent Magnet Synchronous Motor (PMSM), the most convenient approach is to consider a synchronous rotating reference frame that is fixed to the rotor (d–q–0 frame). The transformation of the state variables (voltages, currents, and flux linkages) from the abc stationary reference frame to the rotating dq0 reference frame is performed using the amplitude-invariant transformation matrix, which is expressed as follows:

$$\begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos(\theta) & \cos(\theta-\frac{2\pi}{3}) & \cos(\theta+\frac{2\pi}{3}) \\ -\sin(\theta) & -\sin(\theta-\frac{2\pi}{3}) & -\sin(\theta+\frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \quad (1)$$

Here, θrepresents the angle between the stator phase a and the rotor flux linkage.

Conversely, the variables in the abc stationary reference frame can be obtained from the rotating dq0 components by applying the inverse amplitude-invariant transformation matrix, which is expressed as follows:

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 1 \\ \cos(\theta-\frac{2\pi}{3}) & -\sin(\theta-\frac{2\pi}{3}) & 1 \\ \cos(\theta+\frac{2\pi}{3}) & -\sin(\theta+\frac{2\pi}{3}) & 1 \end{bmatrix}\begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix} \quad (2)$$

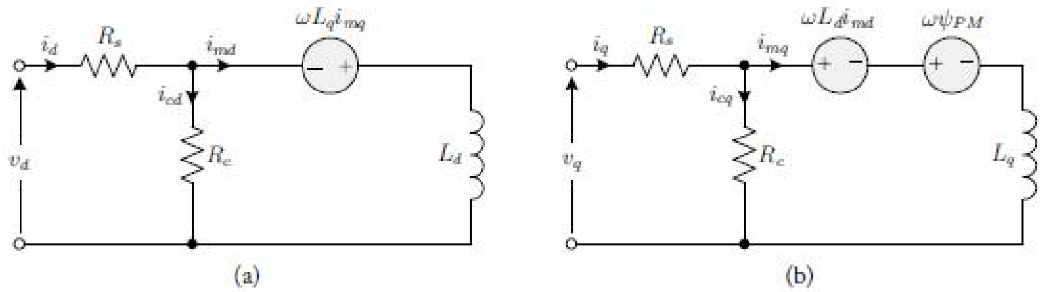

Figure 1. dq-axis equivalent circuits of the PMSM model considering iron losses:
(a) equivalent circuit for the d-axis;
(b) equivalent circuit for the q-axis.

Taking this into account, and assuming that magnetic saturation is neglected, the electromotive force (EMF) is sinusoidal, and the rotor is cageless, the voltage equations of the PMSM (Permanent Magnet Synchronous Motor) in the synchronous reference frame can be expressed as follows:

$$v_d = R_s i_d + L_d \frac{di_{md}}{dt} - \omega L_q i_{mq} \quad (3)$$

$$v_q = R_s i_q + L_q \frac{di_{mq}}{dt} + \omega L_d i_{md} + \omega \psi_{PM} \quad (4)$$

From equations (3) and (4), the state-space equations that allow constructing the complete dynamic model of the PMSM can be derived as follows:

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$$\frac{di_{md}}{dt}=\frac{1}{L_d}(v_d-R_s i_d+\omega L_q i_{mq}) \quad (5)$$

$$\frac{di_{mq}}{dt}=\frac{1}{L_q}(v_q-R_s i_q-\omega L_d i_{md}-\omega\psi_{PM}) \quad (6)$$

$$\frac{d\omega_r}{dt}=\frac{1}{J}(T_e-D\omega_r-T_L) \quad (7)$$

$$\frac{d\theta}{dt}=\omega=\omega_r p \quad (8)$$

Where:

$$i_d=\frac{1}{R_c}(L_d\frac{di_{md}}{dt}-\omega L_q i_{mq}+R_c i_{md}) \quad (9)$$

$$i_q=\frac{1}{R_c}(L_q\frac{di_{mq}}{dt}+\omega L_d i_{md}+\omega\psi_{PM}+R_c i_{mq}) \quad (10)$$

$$i_{cd}=i_d-i_{md}; i_{cq}=i_q-i_{mq} \quad (11)$$

The electromagnetic torque is determined as follows:

$$T_e=\frac{3}{2}p[\psi_{PM}i_{mq}+(L_d-L_q)i_{md}i_{mq}] \quad (12)$$

By considering equations (5), (6), (9), and (10) under steady-state operating conditions, the copper (Cu) losses in the stator windings can be determined using the following expression:

$$P_{Cu}=\frac{3}{2}R_s(i_d^2+i_q^2) \quad (13)$$

Similarly, the iron losses caused by the fundamental component of the total flux linkage in the iron core can be determined as follows: $P_{Fe}=\frac{3}{2}R_c(i_{cd}^2+i_{cq}^2)=\frac{3}{2}\frac{\omega^2}{R_c}[(L_q i_{mq})^2+(\psi_{PM}+L_d i_{md})^2]$ (14)

Although hysteresis losses are not considered in this study, they can also be included in the machine model. These losses are proportional to the frequency of the machine phase currents. Therefore, to incorporate them into the model, the iron-loss resistance (R_{Fe}) is usually treated as a function of the angular frequency (ω).

Summary. The research results indicate that modeling a PMSM in the dq coordinate system simplifies the electromechanical processes of the motor and allows for more accurate calculations. Representing iron losses through the Rc resistance increases the realism of the model and plays a significant role in analyzing energy efficiency. Using the mathematical model of the PMSM, it is possible to evaluate the motor's dynamic performance, power losses, and the efficiency of control systems with high accuracy. This approach serves as an effective tool for enhancing the energy efficiency of industrial enterprises and developing new control algorithms based on digital simulations.

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