

## METHODS OF TEACHING THE TOPIC “POLYGONS” IN ACADEMIC LYCEUMS

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**Abstract:** One of the most important issues for academic lyceums is to form a person who can carry out their actions independently without the help of others, has their own opinion, and can objectively evaluate and control the results of their own activities.

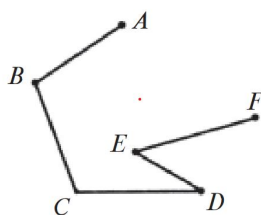
The main issue of academically oriented lyceums is to provide students with the ability to work independently.

In the classes conducted in academic lyceums, students' independent work is a conditional component of the educational process. Its role, content, and duration are purposefully defined in the study of the material, while its specificity and level are determined by the level of preparation of lyceum students.

**Key words:** broken line, diagonal, interior angle, closed region, convex, regular, similar polygon, segment.

In this article, information on the topic “**Polygons**” is not presented to cover the topic fully or to solve all problems. Rather, the aim is to assist students in choosing the correct approach to solving such problems by providing examples of solution methods for some frequently encountered problems in entrance examinations that seem difficult and complex for applicants, are solved using non-standard methods, and require logical reasoning.

**Definition 1.** A shape formed by line segments **AB, BC, CD, DE, EF**, where the end of one segment is consecutively connected to the beginning of the next one, is called a broken line **ABCDEF** (Figure 1).



In this case, the points **A, B, C, D, E, F** are called the vertices of the broken line, the segments **AB, BC, CD, DE, EF** are its parts, and the points **A** and **F** are called the ends of the broken line.

Figure 1

If none of the three points of a broken line lie on the same straight line and none of its segments intersect at interior points, it is called a simple broken line (Figure 2a).

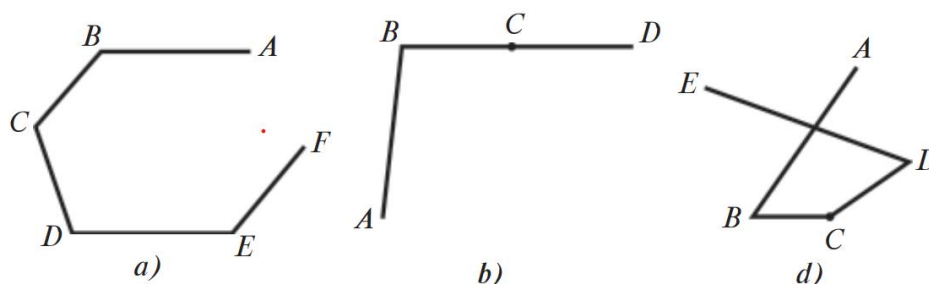
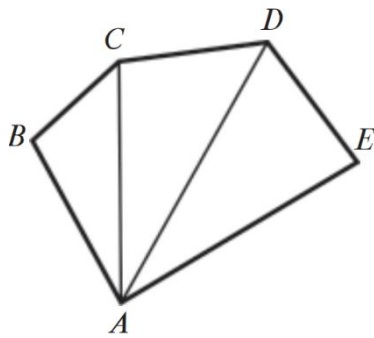


Figure 2

The sum of the lengths of the segments of a broken line is called its perimeter. Obviously, the perimeter of the broken line  $ABCDE$  is not smaller than the distance  $AE$  between its endpoints (Figure 3).



Indeed, by connecting one vertex of the broken line with the segments opposite to it, we form the triangles  $\triangle ABC$ ,  $\triangle ACD$ , and  $\triangle ADE$ .

Applying the triangle inequality to each of these triangles, we obtain the relations

$AB + BC > AC$ ,  $AC + CD > AD$ , and  $AD + DE > AE$ . Since the

obtained inequalities are of the same type, they can be added term by term:

$AB + BC + AC + CD + AD + DE > AC + AD + AE$ .

Figure 3

By simplifying like terms, we obtain the required inequality  $AB + BC + CD + DE > AE$ .

If the endpoints of a broken line coincide, it is called a closed broken line.

**Definition 2.** A part of the plane bounded by a simple closed broken line is called a polygon. The vertices and segments of the broken line are called the vertices and sides of the polygon, respectively. The polygon with the smallest number of sides is a triangle. The name of a polygon is given according to the number of its sides.

**Definition 3.** A segment that connects two vertices of a polygon that do not lie on the same side is called its diagonal. A triangle has no diagonals, while a quadrilateral has two diagonals.

**Definition 4.** The sum of the lengths of all sides of a polygon is called its perimeter. Any polygon divides the plane into two parts: the part bounded by the sides of the polygon is called the interior region of the polygon, and the part lying outside the polygon is called its exterior region. Let the polygon  $ABCDE$  be given. We extend any one of its sides, for example  $BC$  (Figure 4a). If the polygon lies on one side of the straight line  $BC$ , it is called a convex polygon.

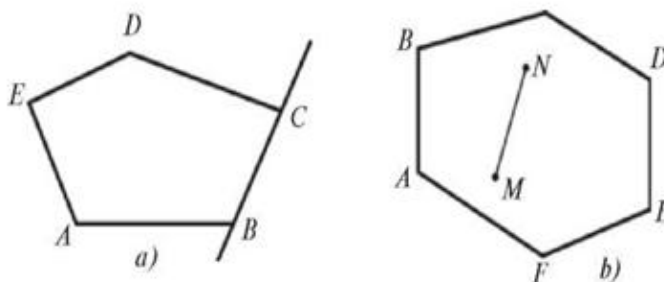


Figure 4

In a convex polygon, any segment  $MN$  connecting two arbitrary points  $M$  and  $N$  inside its interior region lies entirely within that region (Figure 4b). However, in the polygon shown in Figure 5, the segment  $PQ$  connecting the interior points  $P$  and  $Q$  lies partly in the interior region and partly in the exterior region of the polygon. For this reason, the polygon  $A_jB_jC_jD_jE_j$  is called a non-convex polygon. If, for example, we extend the side  $C_1D_1$  of the non-convex polygon  $A_jB_jC_jD_jE_j$ , it is divided into two polygons that lie on different sides of the straight line  $C_1D_1$ .

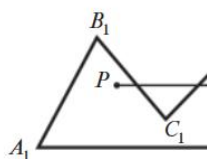
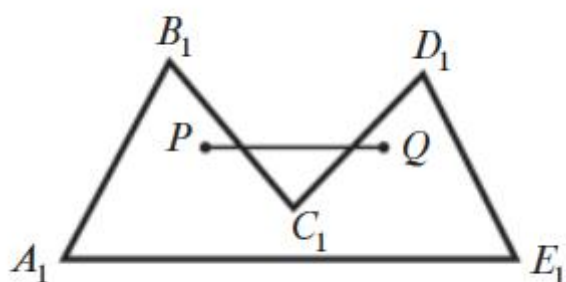


Figure 5

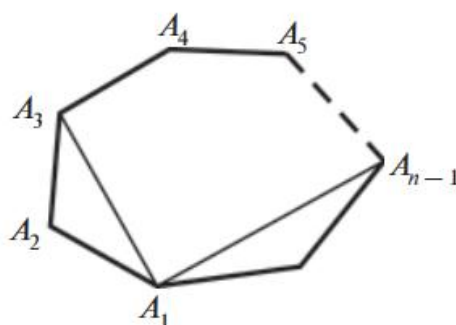


Figure 6

**Theorem 1.** The sum of the interior angles of a convex  $n$ -gon is equal to  $180^\circ (n - 2)$ .

**Proof.** Let a convex  $n$ -gon  $A_1 A_2 \dots A_n$  be given. We connect one of its vertices, for example  $A_1$  with the remaining vertices and form triangles (Figure 6). Each of the triangles  $A_1 A_2 A_3$  and  $A_1 A_{n-1} A_n$  is formed by two sides of the given polygon, while each of the remaining triangles contains only one side of the polygon. Therefore, the total number of triangles formed is  $n - 2$ . Since the sum of the interior angles of a triangle is  $180^\circ$ , the sum of the interior angles of a convex  $n$ -gon is  $180^\circ (n - 2)$ . The theorem is proved.

- 1) From one vertex of a polygon,  $(n-3)$  diagonals can be drawn;
- 2) The diagonals drawn from one vertex divide the polygon into  $(n-2)$  triangles;
- 3)  $\alpha_1 + \alpha_2 + \dots + \alpha_n = 180(n-2)$ ;
- 4) Exterior angles:  $\beta_1 + \beta_2 + \dots + \beta_n = (180^\circ - \alpha_1) + (180^\circ - \alpha_2) + \dots + (180^\circ - \alpha_n) = 360^\circ$ ;
- 5) Convex polygon:  $0^\circ < \alpha < 180^\circ$ ,  $0^\circ < \beta < 180^\circ$ ;
- 6) Number of diagonals:  $\frac{n(n-3)}{2}$ .

**Problem.** The sum of the interior angles of a convex polygon and one exterior angle is  $1940^\circ$ . 1)  $n$ ? 2)  $\alpha$ ?

**Solution.**  $0^\circ < \beta < 180^\circ$ ,  $180(n-2) + \beta = 1940^\circ$

$$\beta = 1940^{\circ} - 180n + 360^{\circ} = 2300^{\circ} - 180n$$

$$0 < 2300^{\circ} - 180n < 180^{\circ}$$

$$\begin{cases} 180n < 2300 \\ 180n > 2120 \end{cases} \Rightarrow \begin{cases} n < \frac{115}{9} \\ n > \frac{106}{9} \end{cases} \Rightarrow \begin{cases} n < 12\frac{7}{9} \\ n > 11\frac{7}{9} \end{cases} \Rightarrow n=12, \beta=140^{\circ}, \alpha=40^{\circ}.$$

**Regular Polygons**

**Definition 5.** If a convex polygon has: a) all sides equal; b) all interior angles equal, then it is called a regular polygon.

We have proved above that the sum of the interior angles of a polygon is equal to  $180^{\circ}(n-2)$ . Therefore, the interior angle of a regular polygon

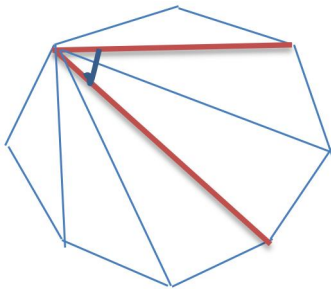
1)  $\alpha = \frac{180^{\circ}(n-2)}{n}$ ,  $\alpha$  - interior angle;

2)  $\beta = \frac{360^{\circ}}{n}$ ,  $\beta$  - exterior angle

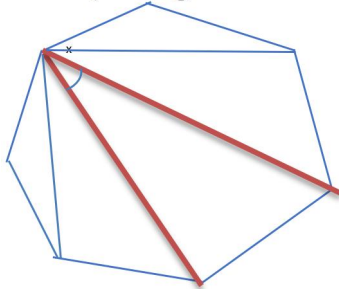
3)  $\varphi = \frac{\frac{180(n-2)}{n}}{n-2} = \frac{180}{n}$

4)  $n$  is even (the angle between the longest and the shortest diagonals)

$$\gamma = \frac{180^{\circ}(n-2)}{2n} - \frac{180^{\circ}}{n} = \frac{180(n-4)}{2n}$$

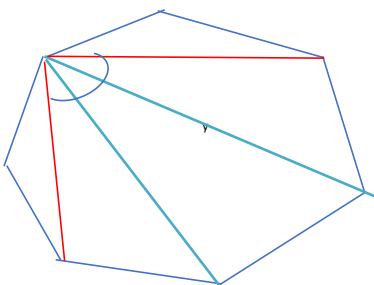


5)  $n$  is odd (the angle between the longest diagonals)

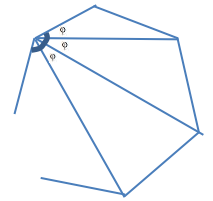


$$x = \frac{180^{\circ}}{n}$$

6)  $n$  is arbitrary (the angle between the shortest diagonals)



$$y = \frac{180^{\circ}(n-2)}{n} - \frac{180^{\circ}}{n} - \frac{180^{\circ}}{n} = \frac{180^{\circ}(n-4)}{n}$$



### Regular hexagon

For a regular hexagon:

- the sum of its interior angles is  $720^{\circ}$ ;
- one interior angle is  $120^{\circ}$ ;
- the sum of the distances (perpendiculars) from an arbitrary interior point to its sides is equal to  $3h$ :  $h_1+h_2+h_3+h_4+h_5+h_6=3h$  (derived on the board);
- the shorter diagonal of a regular hexagon with side length  $a$  is equal to  $a\sqrt{3}$  (derived on the board);
- the longer diagonal of a regular hexagon with side length  $a$  is equal to  $2a$  (derived on the board);
- the area of a regular hexagon with side length  $a$  is  $S=\frac{a^23\sqrt{3}}{2}$  (derived on the board).

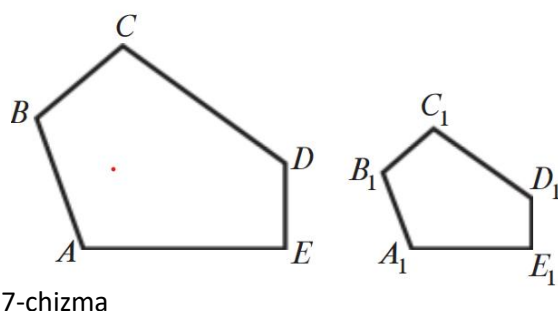
### Similar Polygons

**Definition 7.** Two polygons are called similar if:

- 1) their corresponding angles are equal; 2) their corresponding sides are proportional.

**Theorem 3.** The perimeters of similar polygons are in the same ratio as their corresponding sides.

**Proof.** Let the polygons  $ABCDE$  and  $A_1B_1C_1D_1E_1$  be similar (Figure 7). From the definition of similar polygons



$$\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{CD}{C_1D_1} = \frac{DE}{D_1E_1} = \frac{EA}{E_1A_1}.$$

Using the properties of proportions (the golden rule), when several equal ratios are given, the ratio of the sum of all antecedent terms to the sum of all consequent terms is equal to the ratio of any one antecedent to its corresponding consequent. That is,

$$\frac{AB+BC+CD+DE+EA}{A_1B_1+B_1C_1+C_1D_1+D_1E_1+E_1A_1} = \frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{CD}{C_1D_1} = \frac{DE}{D_1E_1} = \frac{EA}{E_1A_1} = k$$

Here,  $k$  is called the coefficient of similarity.

**Theorem 4.** Similar polygons can be divided into the same number of similar and similarly positioned triangles.

**Proof.** Let two similar polygons  $ABCDE$  and  $A_1B_1C_1D_1E_1$  be given. Inside the first polygon  $ABCDE$ , choose an arbitrary point  $O$  and connect it with the vertices  $A, B, C, D, E$  of the polygon. As a result, the polygon is divided into as many triangles as the number of its sides (Figure 8). On the side  $A_1E_1$  of the polygon  $A_1B_1C_1D_1E_1$  construct two angles:  $\angle O_1A_1E_1 = \angle OAE$  and  $\angle O_1E_1A_1 = \angle OEA$ . It is known that the segments  $O_1E_1$  and  $O_1A_1$  intersect at the point  $O_1$ . Then, by construction,  $\triangle AOE \sim \triangle A_1O_1E_1$ .

Now let us prove the similarity of  $\triangle AOE$  and  $\triangle A_1 O_1 E_1$ . From the similarity of the polygons, it follows that

$$\angle BAE = \angle B_1 A_1 E_1 \text{ and } \frac{AB}{A_1 B_1} = \frac{EA}{E_1 A_1} \quad (1)$$

Since  $\triangle AOE \sim \triangle A_1 O_1 E_1$ , we have

$$\angle OAE = \angle O_1 A_1 E_1 \text{ and } \frac{AO}{A_1 O_1} = \frac{AE}{A_1 E_1} \quad (2)$$

From equalities (1) and (2), we obtain

$$\angle BAO = \angle B_1 A_1 O_1 \text{ and } \frac{BA}{B_1 A_1} = \frac{AO}{A_1 O_1}$$

Therefore,  $\triangle ABO \sim \triangle A_1 B_1 O_1$ . Similarly, it can be proved that  $\triangle COD \sim \triangle C_1 O_1 D_1$  and so on. In this case, it is known that similar triangles are similarly positioned.

**Theorem 5.** The ratio of the areas of similar polygons is equal to the ratio of the squares of their corresponding sides.

(The proof is given to students as homework.)

### Problems Related to the Topic

1. The sum of the exterior angles and one interior angle of a convex pentagon is  $400^\circ$ . If the remaining interior angles are in the ratio  $1 : 2 : 3 : 4$ , find all interior angles and their arithmetic mean.
2. The smallest interior angle of a convex polygon is  $60^\circ$ , and its interior angles increase by  $20^\circ$  each time:  $60^\circ, 80^\circ, 100^\circ, \dots$ . What is the maximum possible number of sides of this polygon?
3. The number of diagonals of a convex  $n$ -gon is not less than **25** and not more than **30**. What values can  $n$  take?
4. If the number of diagonals of a convex polygon is **twice** the number of its sides, how many sides does the polygon have?
5. What are the interior and exterior angles of a regular hexagon?
6. How many diagonals are drawn from one vertex of a regular polygon if one interior angle is  $160^\circ$ ?
7. What is the measure of the central angle of a regular **15-gon**?

### Conclusion

As is known, the future of any society is determined by the level of development of its education system, which is an inseparable part of that society. Modern ways of presenting information include technological knowledge and widely use diagrams, pictures, tables, graphs, and charts. This approach significantly activates students' visual memory and strengthens their ability to remember information.

This article discusses the methodology of teaching the topic "**Polygons**" in the geometry course of academic lyceums. Using various methods for solving geometry problems increases the effectiveness of lessons. A creative approach to the content of problems, as well as the construction of new problems and their solutions, allows the use of innovative technologies, information technology tools, and interactive teaching methods, which expand students' thinking skills. As a result, students' interest in learning geometry is formed and strengthened.

While writing this article, I collected many valuable, interesting, and worthwhile materials for my own learning. The topics are presented briefly and clearly, and the problems given to reinforce the topic are well matched to the content. The quality and speed of independent work depend on how it is assigned; therefore, we must take into account students' problem-solving levels and the difficulty of the tasks.

For this reason, this article presents solutions to a number of geometry problems related to polygons that are encountered by academic lyceum students. Organizing lessons in general secondary schools and academic lyceums using new pedagogical technologies is a requirement of our time. It is important to select these technologies appropriately according to the topic.

In conclusion, it can be said that this article can be widely used. I hope it helps to form a broader understanding of teaching methodology through the use of innovative technologies in lessons.

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