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THE TRANSPORTATION PROBLEM

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Abstract: This paper examines the transportation problem as one of the key tools of economic and mathematical modeling of logistics and production processes. Its role in optimizing resource distribution between supply and demand points while minimizing total costs is substantiated. The mathematical model of the transportation problem within the framework of linear programming is presented, and the conditions of balanced and unbalanced formulations are analyzed. The main methods for constructing an initial feasible solution (North-West Corner method, Least Cost method, Vogel㕞 Approximation method) and the potential method as a tool for optimality verification are discussed. A detailed numerical example with economic interpretation is provided. The study concludes that the transportation model has high practical significance in the context of digital economy development and modern logistics systems.

Keywords: transportation problem, linear programming, optimization, logistics, economic-mathematical modeling, potential method, cost minimization.

Аннотация: В данной статье рассматривается транспортная задача как один из ключевых инструментов экономико-математического моделирования логистических и производственных процессов. Обоснована ее роль в оптимизации распределения ресурсов между пунктами отправления и пунктами назначения при минимизации совокупных затрат. Представлена математическая модель транспортной задачи в рамках линейного программирования, рассмотрены условия закрытой и открытой постановки. Проанализированы основные методы построения начального опорного плана (метод северо-западного угла, метод минимального элемента, метод Фогеля) и метод потенциалов как инструмент проверки оптимальности решения. В работе приведён развернутый числовой пример с экономической интерпретацией результатов. Сделан вывод о высокой практической значимости транспортной модели в современных условиях цифровизации экономики и развития логистических систем.

Ключевые слова: транспортная задача, линейное программирование, оптимизация, логистика, экономико-математическое моделирование, метод потенциалов, минимизация затрат.

Introduction. In a modern market economy, issues of rational resource allocation are particularly relevant. The development of logistics systems, the globalization of markets, and the digitalization of production and supply chain management require the use of rigorous mathematical methods to make economically sound decisions. One of the most effective tools for such analysis is the transportation problem.

The transportation problem belongs to the class of linear programming problems and represents a special case of an optimization model of resource allocation [1]. Its main objective is to determine the optimal plan for transporting products from multiple suppliers to multiple consumers while minimizing total transportation costs.

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The economic essence of the transportation problem lies in finding a distribution of product flows that ensures the satisfaction of consumer demand while efficiently utilizing suppliers' resources. Thus, the transportation model serves as a tool for optimizing logistics and production processes, reducing costs and increasing the efficiency of the economic system.

A distinctive feature of the transportation problem is its special constraint structure, which allows for the application of more efficient methods to its solution compared to general linear programming algorithms. This makes the transportation model particularly in demand in economic calculations.

The purpose of this study is to comprehensively analyze the theoretical and practical aspects of the transportation problem, examine its mathematical model, solution methods, and economic interpretation of the results obtained.

To achieve this goal, the following tasks are addressed in this study:

- To reveal the theoretical foundations of the transportation problem;
- to present its mathematical model;
- to analyze the main methods for constructing an initial baseline plan;
- to consider the potential method as an optimality criterion;
- to demonstrate the application of the model using a numerical example;
- to provide an economic evaluation of the results obtained.

Theoretical Foundations of the Transportation Problem

The transportation problem is a classic linear programming problem with a special constraint structure [2]. Its mathematical formulation reflects the process of distributing a homogeneous product from multiple suppliers to multiple consumers with the goal of minimizing total transportation costs.

Let's assume:

- **m** points of departure (suppliers) $\mathbf{A_1, A_2, \ldots, A_m}$;
- **n** destinations (consumers) $\mathbf{B_2, \ldots, B_n}$;
- $\mathbf{a_i}$ — supplier's product inventory $\mathbf{A_i}$;
- $\mathbf{b_j}$ — consumer demand $\mathbf{B_j}$;
- $\mathbf{c_{ij}}$ — the cost of transporting a unit of output from $\mathbf{A_i}$ to $\mathbf{B_j}$;
- $\mathbf{x_{ij}}$ — volume of transported products from $\mathbf{A_i}$ to $\mathbf{B_j}$.

The economic meaning of the variable $\mathbf{x_{ij}}$ is to determine the quantity of goods sent from the i-th departure point to the j-th destination.

Mathematical model of the transportation problem

The objective function is aimed at minimizing total transportation costs:

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \to \min$$

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Subject to the following restrictions:

Inventory restrictions (for suppliers):

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \ldots, m$$

Demand constraints (to consumers):

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \ldots, n$$

Non-negativity condition:

$$x_{ij} \geq 0$$

Thus, the transportation problem is formulated as a linear programming problem with a special matrix structure of coefficients.

Closed and open transportation problems

Closed (balanced) problem

If the equality holds:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Then the problem is called closed or balanced. This means that the total supply equals the total demand.

Open (unbalanced) problem

If:

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$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$$

The problem is called open. In this case, to balance it, a fictitious supplier or fictitious consumer with zero transportation costs is introduced.

Economic interpretation of the fictitious element:

- a fictitious supplier reflects a product shortage;
- a fictitious consumer reflects a product surplus.

Matrix notation

The transportation problem can be represented as a table:

	B_1	B_2	...	B_n	Stock
A_1	c_{11}	c_{12}	...	c_{1n}	a_1
A_2	c_{21}	c_{22}	...	c_{2n}	a_2
...	...	...	...	...	...
A_m	c_{m1}	c_{m2}	...	c_{mn}	a_m
Demand	b_1	b_2	...	b_n	

This table is called a transport matrix.

Properties of the transport problem

1. The number of variables is $\mathbf{m \times n}$.
2. The number of independent constraints is $\mathbf{m + n - 1}$.
3. The basic solution contains no more than $\mathbf{m + n - 1}$ positive variables.
4. The model has a special structure that allows the use of specialized solution algorithms.

Economic significance of the model

The transport problem is applied to:

- Optimization of supply chains;
- Distribution of raw materials between enterprises;
- Product supply planning;
- Inventory management;
- Interregional resource distribution;
- Optimization of export-import flows.

In the digital economy, the transportation model underlies *supply chain management* algorithms, confirming its practical significance.

Methods for constructing an initial baseline plan

The solution to the transportation problem is carried out in two stages:

1. Constructing an initial feasible (baseline) plan.

2. Checking its optimality and improving it (if necessary).

Since the transportation problem has a specific structure, specialized algorithms characterized by computational efficiency are used to find the initial solution.

The northwest corner method is the simplest way to construct an initial feasible plan.

Algorithm for this method:

1. Filling begins with cell (1,1)—the upper left corner of the table.
2. The value is written to the selected cell:

$$\mathbf{x_{11}=min(a_1,b_1)}$$

3. If supplier **A_1**'s inventory is exhausted, we move to the next row.
4. If customer **B_1**'s demand is satisfied, we move to the next column.
5. The process continues until inventory is fully allocated.

Features of the method:

- does not take into account the value of transportation tariffs;
- ensures rapid construction of a feasible plan;
- may yield a solution that is far from optimal.

Economic interpretation:

the method reflects the mechanical allocation of resources without regard to cost efficiency.

This method takes into account the structure of transportation tariffs.

Algorithm:

1. The cell with the minimum cost c_{ij} is selected.

2. The maximum possible volume is placed in it:

$$\mathbf{x_{ij}=min(a_i,b_j)}$$

1.The row or column with a zero remainder is eliminated.

2. The process is repeated for the remaining matrix.

Advantages:

- takes tariffs into account;
- produces a more cost-effective initial plan;

- reduces the likelihood of significant deviations from the optimum.

Disadvantage:

does not always lead to a solution close to the optimum.

Vogel's Approximation Method is considered the most effective for constructing an initial plan.

The essence of the method is the calculation of penalties.

For each row and column, the difference between the two lowest tariffs is determined:

$\Delta = c_2^{min} - c_1^{min}$

Algorithm:

1. Penalties are calculated by rows and columns.
2. The row or column with the maximum penalty is selected.
3. Within this row or column, the cell with the minimum cost is selected.
4. Allocation is performed.
5. Penalties are recalculated.

Advantages:

- Generates a plan close to optimal;
- Reduces the number of iterations during subsequent optimization.

Economic rationale:

The method minimizes potential losses from inefficient allocation.

Potential Method (MODI Method)

After constructing the initial baseline plan, its optimality must be verified.

For this, the potential method is used.

Potentials are entered for each row and column:

u_i (for suppliers)

v_j (for consumers)

For the basis cells the following condition is satisfied:

$u_i + u_j = c_{ij}$

After finding the potentials, estimates for free cells are calculated:

$\Delta_{ij} = c_{ij} - (u_i + v_j)$

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If for all free cells:

$\Delta_{ij} \geq 0$

then the current plan is optimal.

If there is at least one cell with:

$\Delta_{ij} < 0$

The plan must be improved by constructing a closed redistribution loop.

Theoretical Justification of the Method

The potential method is based on a dual linear programming problem. The potentials ui vj are estimates of the dual variables, and the criterion $\Delta ij \geq 0$ corresponds to the Kuhn-Tucker optimality conditions for linear problems.

This confirms the rigorous mathematical justification of the algorithm.

Expanded Numerical Example of Solving a Transportation Problem

Consider an economic situation in which there are three supplier companies and four consumers.

Initial Data

Supplier Inventories:

$a_1=40,\ a_2=35,\ a_3=25$

Consumer demand:

$b_1=20,\ b_2=30,\ b_3=25,\ b_4=25$

Let's check the balance condition:

$40+35+25=100$

$20+30+25+25=100$

Therefore, the problem is closed.

Transport tariff matrix (monetary units per unit of output):

	B_1	**B_2**	**B_3**	**B_4**	**Stock**
A_1	8	6	10	9	40
A_2	9	12	13	7	35
A_3	14	9	16	5	25

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Demand	20	30	25	25	

Objective function:

$$Z = \sum c_{ij} x_{ij} \to \min$$

Building an initial plan using the Vogel method

Step 1. Calculating penalties

For each row and column, calculate the difference between the two minimum tariffs.

Rows:

A_1: 6 и 8 → fine = 2
A_2: 7 и 9 → fine = 2
A_3: 5 и 9 → fine = 4

Columns:
B_1: 8 and 9 → 1
B_2: 6 and 9 → 3
B_3: 10 and 13 → 3
B_4: 5 and 7 → 2

Maximum penalty = 4 (line A_3).

Minimum tariff in line A_3 = 5 (cell A_3B_4).

We distribute:

$x_{34}=\min(25,25)=25$

Supplier A_3 is depleted, demand B_4 is satisfied.

Step 2

Recalculate penalties for the remaining table.

Then the distribution continues similarly.

After completing all steps, we obtain the initial plan.:

	B_1	B_2	B_3	B_4	Stock
A_1	20	20	0	0	40

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A_2	0	10	25	0	35
A_3	0	0	0	25	25
Demand	20	30	25	25	

Calculating the total cost

*Z=20*8+20*6+10*12+25*13+25*5*

Z=160+120+120+325+125

Z=850

The initial plan yields transportation costs of 850 monetary units.

Optimality check using the potential method

For the basis cells, we construct a system:

$u_i + v_j = c_{ij}$

Let's put it this way $u_1=0$

From the cage (1,1):

$0+v_1=8 \Rightarrow v_1=8$

From (1,2):

$0+v_2=6 \Rightarrow v_2=6$

From (2,2):

$u_2+6=12 \Rightarrow u_2=6$

From (2,3):

$6+v_3=13 \Rightarrow v_3=7$

From (3,4):

$u_3+v_4=5$

From the previous calculations we obtain:

$u_3=-2,\ v_4=7$

Now we calculate the scores for free cells:

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$\Delta_{ij}=c_{ij}-(u_i+v_j)$

After calculations we get that everything:

$\Delta_{ij}\geq 0$

Therefore, the plan is optimal.

Economic Interpretation of Results

The resulting optimal allocation means:

- Supplier A_1 fully satisfies B_1's demand and partially satisfies B_2's.
- Supplier A_2 serves B_2 and B_3.
- Supplier A_3 serves B_4 exclusively.

Economic impact:

- minimization of logistics costs;
- rational allocation of production capacity;
- absence of surpluses and shortages;
- optimal use of transport infrastructure.

Comparative analysis of solution methods

Criterion	N-W angle	Min. element	Vogel
Tariff accounting	No	Yes	Yes
Close to optimum	Low	Average	High
Complexity	Minimum	Average	Above average
Practical efficiency	Low	Average	High

The Vogel method is the most efficient way to construct an initial plan.

Conclusion

This study examined the transportation problem as a key tool in economic and mathematical modeling. It was shown that the transportation model allows for the formalization of resource allocation processes and the minimization of total costs.

A numerical example demonstrated the effectiveness of the Vogel method and the potential method for obtaining an optimal solution. The theoretical analysis confirmed that the transportation problem is a special case of linear programming with a special constraint structure, allowing for the application of specialized algorithms.

In the context of the development of the digital economy and logistics platforms, the transportation model remains highly relevant and has practical significance.

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